

# C2 Exercise 11A (definite integrals)

Note Title

15/05/2011

$$\begin{aligned} 1a \quad & \int_1^2 \left( \frac{2}{x^3} + 3x \right) dx \\ &= \int_1^2 (2x^{-3} + 3x) dx \\ &= \left[ \frac{2}{-2} x^{-2} + \frac{3}{2} x^2 \right]_1^2 \\ &= \left[ -x^{-2} + \frac{3}{2} x^2 \right]_1^2 \\ &= \left( -(2)^{-2} + \frac{3}{2} (2)^2 \right) - \left( -(1)^{-2} + \frac{3}{2} (1)^2 \right) \\ &= -\frac{1}{4} + 6 + 1 - \frac{3}{2} \\ &= 5\frac{1}{4} \end{aligned}$$

1b

$$\int_0^2 (2x^3 - 4x + 5) dx$$

$$= \left[ \frac{2}{4}x^4 - \frac{4}{2}x^2 + 5x \right]_0^2$$

$$= \left( \frac{1}{2} \times 16 - 2 \times 4 + 5 \times 2 \right) - (0)$$

$$= 10.$$

1c

$$\int_4^9 \left( \sqrt{x} - \frac{6}{x^2} \right) dx$$

$$= \int_4^9 \left( x^{1/2} - 6x^{-2} \right) dx$$

$$= \left[ \frac{2}{3} x^{3/2} + 6x^{-1} \right]_4^9$$

$$= \left( \frac{2}{3} (\sqrt{9})^3 + \frac{6}{9} \right) - \left( \frac{2}{3} (\sqrt{4})^3 + \frac{6}{4} \right)$$

$$= 18 + \frac{2}{3} - \frac{16}{3} - \frac{3}{2}$$

$$= 18\frac{2}{3} - 5\frac{1}{3} - \frac{3}{2}$$

$$= 13\frac{1}{3} - 1\frac{1}{2}$$

$$= 11\frac{5}{6}$$

$$1d \quad \int_1^2 \left( 6x - \frac{12}{x^4} + 3 \right) dx$$

$$= \int_1^2 (6x - 12x^{-4} + 3) dx$$

$$= \left[ 3x^2 + 4x^{-3} + 3x \right]_1^2$$

$$= \left( 12 + \frac{1}{2} + 6 \right) - (3 + 4 + 3)$$

$$= 18\frac{1}{2} - 10$$

$$= 8\frac{1}{2}$$

$$1e \quad \int_1^8 (x^{-1/3} + 2x - 1) dx$$

$$= \left[ \frac{3}{2} x^{2/3} + x^2 - x \right]_1^8$$

$$= \left( \frac{3}{2} (\sqrt[3]{8})^2 + 8^2 - 8 \right) - \left( \frac{3}{2} (\sqrt[3]{1})^2 + 1 - 1 \right)$$

$$= (6 + 64 - 8) - (3/2)$$

$$= 60\frac{1}{2}$$

2a

$$\int_1^3 \frac{x^3 + 2x^2}{x} dx$$

$$= \int_1^3 (x^2 + 2x) dx$$

$$= \left[ \frac{1}{3}x^3 + \frac{2}{2}x^2 \right]_1^3$$

$$= (9 + 9) - \left( \frac{1}{3} + 1 \right)$$

$$= 16\frac{2}{3}.$$

2b

$$\int_1^4 (\sqrt{x} - 3)^2 dx$$

$$= \int_1^4 (x - 6x^{1/2} + 9) dx$$

$$= \left[ \frac{1}{2}x^2 - \frac{6}{3/2}x^{3/2} + 9x \right]_1^4$$

$$= \left[ \frac{1}{2}x^2 - 4x^{3/2} + 9x \right]_1^4$$

$$= \left( \frac{1}{2}(4)^2 - 4(\sqrt{4})^3 + 9(4) \right) - \left( \frac{1}{2}(1)^2 - 4(1) + 9 \right)$$

$$= (8 - 32 + 36) - (5\frac{1}{2})$$

$$= 6\frac{1}{2}$$

2c

$$\int_3^6 \left(x - \frac{3}{x}\right)^2 dx$$

$$= \int_3^6 (x^2 - 6 + 9x^{-2}) dx$$

$$= \left[ \frac{1}{3}x^3 - 6x - 9x^{-1} \right]_3^6$$

$$= \left( \frac{1}{3}(216) - 6(6) - \frac{9}{6} \right) - \left( \frac{1}{3}(27) - 6(3) - \frac{9}{3} \right)$$

$$= (72 - 36 - 1\frac{1}{2}) - (9 - 18 - 3)$$

$$= 34\frac{1}{2} + 12$$

$$= 46\frac{1}{2}$$



$$2d \int_0^1 x^2 \left( \sqrt{x} + \frac{1}{x} \right) dx$$

$$= \int_0^1 (x^{5/2} + x) dx$$

$$= \left[ \frac{2}{7} x^{7/2} + \frac{1}{2} x^2 \right]_0^1$$

$$= \left( \frac{2}{7} + \frac{1}{2} \right) - (0 + 0)$$

$$= \frac{4}{14} + \frac{7}{14}$$

$$= \frac{11}{14}$$

2e

$$\int_1^4 \frac{2 + \sqrt{x}}{x^2} dx$$

$$= \int_1^4 (2x^{-2} + x^{-3/2}) dx$$

$$= \left[ -2x^{-1} - 2x^{-1/2} \right]_1^4$$

$$= \left( \frac{-2}{4} - 2\left(\frac{1}{2}\right) \right) - \left( \frac{-2}{1} - 2(1) \right)$$

$$= -\frac{1}{2} - 1 + 2 + 2$$

$$= 2\frac{1}{2}$$